Cranwell Primary School

Progression in calculation for Multiplication and Division

Vocabulary

Counting, steps, each, doubling, scaling, times, twice as big, count in ones, count in _____, lots of, groups of, x, times, multiply, multiplied by, multiple of, repeated addition, arrays, row, column, double, group in pairs, multiplication, product, inverse.

Remember

Use the language ‘calculation’ not ‘sum’ (sum means plus or total)

Use the language ‘digit’ not number (number is the amount or quantity)

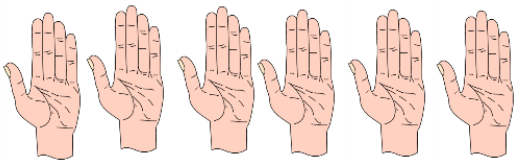
Stage 1

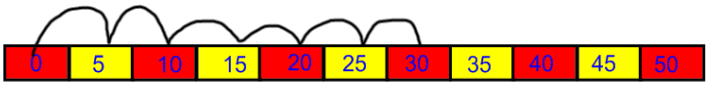
Children will experience practical oppertunities involving equal sets or groups using a wide variety of equipment. Practical resources will support childrens development of mental pictures and images.

Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings (legs on animals and pairs). They will begin to use the language and associated representation of doubling.

Counting in pairs of socks

How many fingers is there altogether on six hands?



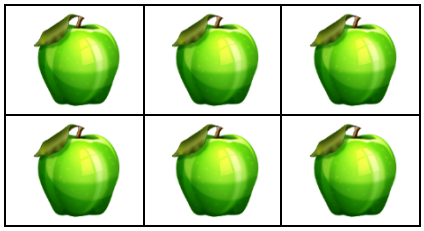


How much money do I have?

are

How

altogether?



There 10 crayons in each box. many crayons are there

Using the language of doubles 3 + 3 =

Stage 2

Children will begin to arrange objects into equal groups to aid counting.

They will continue to count in multiples and begin to realte this to multiplication through finger counting. Children will be introduced to a variety of represenations of repeating addition. They will see the representations alongside each other and begin to make connections between them. (Five fingers on a hand and jumps of five on a number track).

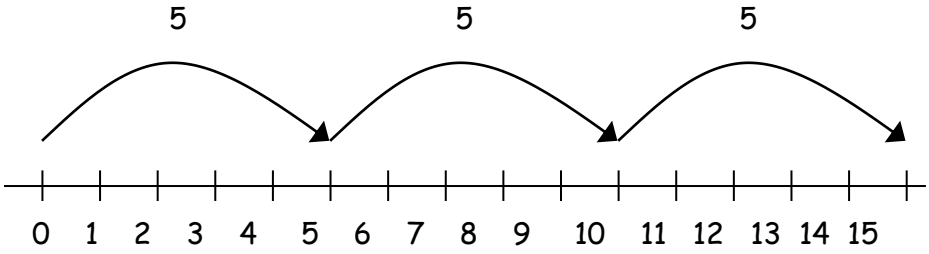
Children will be introduced to the arrays using equipment. They should explore arrays in the world around us e.g egg boxes and use them to answer questions such as 'How many eggs would be needed to fill 3 boxes? How many muffins in 4 baking tins? This will be related to repeated addition.

6 + 6 + 6

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

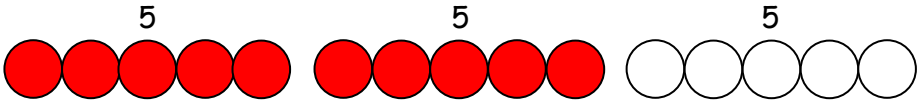
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



and on a bead bar

5 x 3 = 5 + 5 + 5



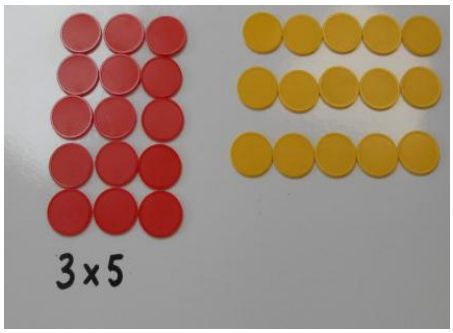
Stage 3

Children will continue to count in multiples and relate this to multiplication through finger counting. They will be able to spot missing numbers in a pattern.

3, 6, 9 ____ 15, 18, ____, 24

Arrays

Children should be able to model a multiplication calculation using an array. This is an effective method of counting and making the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explain what they show. This knowledge will support with the development of the grid method.



Children should be confient with their use of the language of scaling when talking about multiplication i.e

I'm 3 times as tall as you. I'm three metres tall. How tall are you?

Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon

5 cm 20 cm

Using symbols to stand for unknown numbers to complete equations using inverse operations

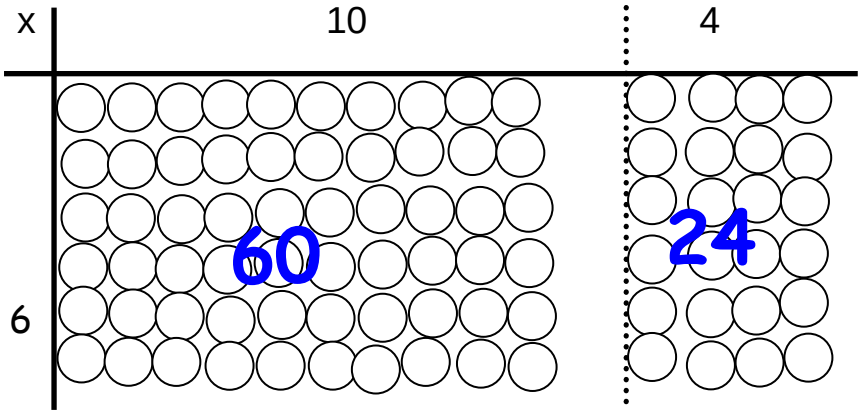
☐ x 5 = 20 3 x △ = 18 ☐ x O = 32

Stage 4

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them effeciently calculate totals e.g. for 7 x 8… Children may find counting in 7s or 8s tricky but they can look for numbers which are easier to calculate e.g 4x5, 4x2, 4x5, 4x2

Stage 5

Children will continue to explore arrays, exporing larger numbers, leading to the grid method of multiplication. Practcal experiences may still be needed. To begin with children should see the array with the grid lines. When appropriate, they should move to using the grid with numbers only.



(6 x 10) + (6 x 4)

60 + 24

84

x	10	4
6	60	24

Stage 6

Children will now be secure in using the grid method for multiplying by one digit numbers and will begin to explore the link between the grid method and the expanded method of short multiplication.

	1	4	
x		6	
	2	4	(6x4)
+	6	0	(6x10)
	8	4	

Explore using arrays and the grid method for multiplying by two digit numbers.

x	10	8
10	100	80
3	30	24

TU x U

24 x 5

x	5
20	100
4	20

= 120

HTU x U 148 x 3

x	3
100	300
40	120
8	24

= 444

TU x TU

36 x 27

x	30	6
20	600	120
7	210	42

= 972

Stage 7

Children will have a good understanding of the expanded short multiplication method and will begin to represent this as compact short multiplication for TU X U.

```
    1 4
x     6
-------
    2 4  (6x4)
+   6 0  (6x10)
-------
    8 4
```

→

```
    1 4
x     6
-------
    8 4
    2
```

Children will be secure in mutiplying two digit numbers and will begin to explore the links between the grid method and the explanded method of long multiplication.

x	10	8
10	100	80
3	30	24

→

```
    1 8
x   1 3
-------
    8 0   (10x8)
    3 0   (3x10)
    2 4   (3x8)
+ 1 0 0   (10x10)
-------
  2 3 4
-------
  1
```

Stage 8

Children will now have a good understanding of the short multiplication method.

Children will now have a good understanding of the expanded long multiplication method and will begin to represent this as compact long multiplication.

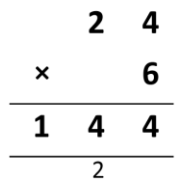
```
    1 8
    1 3
  -------
    2 4     (3x8)
    3 0     (3x10)
    8 0     (10x8)
+ 1 0 0     (10x100)
  -------
  2 3 4
  -------
  1
```

```
      2
    1 8
  x 1 3
  -------
    5 4
+ 1 8 0
  -------
  2 3 4
  -------
  1
```

Summary of the written methods

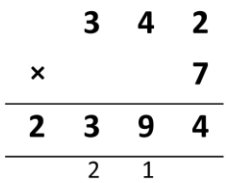
Short multiplication

24 × 6 becomes



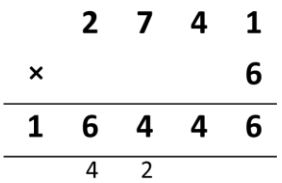
Answer: 144

342 × 7 becomes



Answer: 2394

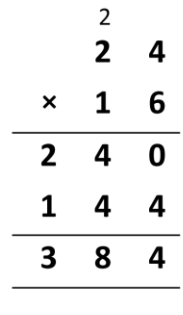
2741 × 6 becomes



Answer: 16 446

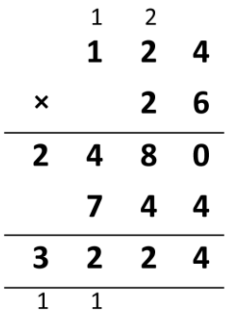
Long multiplication

24 × 16 becomes



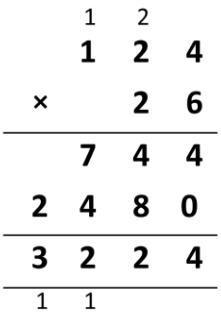
Answer: 384

124 × 26 becomes



Answer: 3224

124 × 26 becomes



Answer: 3224

Division

Vocabulary

Halve, share, share equally, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of.

Stage 1

Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between groups and people. There will be an emphasis on sharing equally.

Children will be introduced to 'grouping' objects. E.g each person will get 2 biscuits.

Children begin to use the language and images of halving. They use pictures to show their findings.

Stage 2

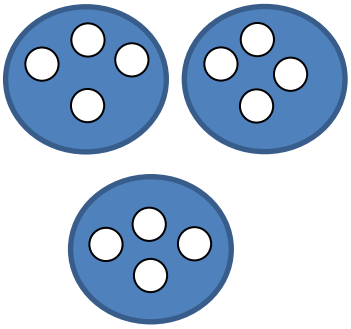
Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line whilst also using equipment.

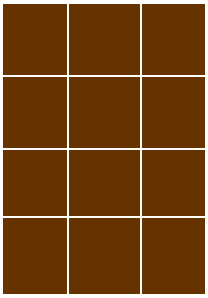
How many 2s are in 10?

How many 2s can we take away from 8?

Children will use their knowledge of counting in multiples to solve division calculations. They will recognise that this is the inverse of multiplication.

Children will begin to group objects in arrays rather than scattered groups





Children make links between division and fractions. They know that the division sign is the equivalent to the fractions.

$1 \div 2 \quad \frac{1}{2}$

Stage 3

Children continue to use knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

They will build on their use of arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

Stage 4

Children continue to organise groups into arrays using larger numbers.

How many in each group? How many groups?

Children should continue to experience the language of scaling. They scale down pictures by dividing by powers of 10)

120 ÷ 3

120 shared equally between 3 is 40

120 shared equally between 4 is 30

3 equal groups of 40 make 120

4 equal groups of 30 make 120

10	10	10	10
10	10	10	10
10	10	10	10

Stage 5

Children will continue to work with concrete arrays, exploring known multiplication/ division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the arrays within short division can be interpreted for both sharing between or equal groups of where the dots within the arrays each represent 1.

56- How many equal groups can I make?

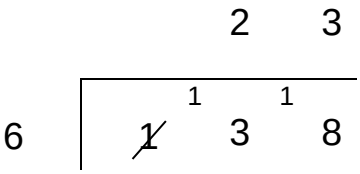
If I put these into 7 equal groups. How many in each group?

Stage 6

Children will work with equipment to divide any integer by a single digit divisor using their knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders. They will recognise that they don't 'fit' into the array.

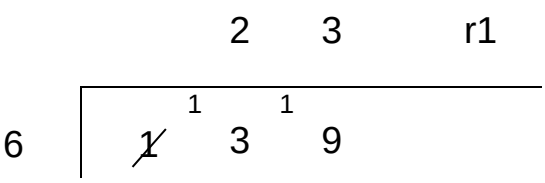
Children are introduced to the notation of short division.



Stage 7

Children will be confident and secure in using short division.

They will now begin to use the short division notation for calculation involving a remainder.



Children use jottings of friendly numbers to support long division of calculations with 2 digit divisors.

Children explore chunks of numbers

1 x 15 = 15
2 x 15 =30
4 x 15 =60
5 x 15 = 75
8 x 15 = 120
10x 15 = 150
20 x 15 = 300
50 x 15 = 750

		2	8	
15	4	2	0	
–	3	0	0	(20 x 15)
	1	2	0	
_	1	2	0	(8 x 15)
			0	

Pencil and paper procedures- Chunking
256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40

* Partition the dividend into multiples of the divisor:

e.g. 256 = 210 + 46

210 ÷ 7 = 30

46 ÷ 7 = 6r4 → 30 + 6r4 = 36r4

OR

```
  256
- 210   (30 groups)
   46
-  42   (6 groups)
    4
```

Stage 8

Children will now be secure using short division for one- digit divisors and long division for two digit divisors with an integer quotient. They will explore long division for two digit divisors which may include a remainder. They will begin to interpret remainders as whole number remainders, fractions or by rounding as appropriate for the context.

1 x 15 = 15
2 x 15 =30
4 x 15 =60 20 x 15 = 300
8 x 15 = 120
10x 15 = 150

		2	8	r12
15	4	3	2	
–	3	0	0	(20 x 15)
	1	3	2	
–	1	2	0	(8 x 15)
		1	2	

Summary of the written methods

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\,9\ 8} \\ \scriptstyle 2 \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 8\ 6\ \text{r}\,2 \\ 5\overline{)\,4\ 3\ 2} \\ \scriptstyle 3 \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 4\ 5\ \text{r}\,1 \\ 1\ 1\overline{)\,4\ 9\ 6} \\ \scriptstyle 5 \end{array}$$

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\ \text{r}\,12 \\ 1\ 5\overline{)\,4\ 3\ 2} \\ 3\ 0\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8 \\ 1\ 5\overline{)\,4\ 3\ 2} \\ 3\ 0\ 0 \quad 15\times20 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \quad 15\times8 \\ \hline 1\ 2 \end{array}$$

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\cdot 8 \\ 1\ 5\overline{)\,4\ 3\ 2\cdot 0} \\ 3\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2\ 0 \\ 1\ 2\ 0 \\ \hline 0 \end{array}$$

Answer: 28·8